Practice 3

Topic: ***Research ACS on observability by R. Kallman and E. Gilbert's criteria***

Example. The dynamic system is described in state-space by the system of the equations:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}, \qquad (*)$$

where $A = \begin{vmatrix} -4 & 5 \\ 1 & 0 \end{vmatrix}$, $B = \begin{vmatrix} -5 \\ 1 \end{vmatrix}$, $C = \begin{vmatrix} 1 & -1 \end{vmatrix}$.

Check the researched dynamic system for observability by R. Kallman and E. Gilbert's criteria.

Algorithm and solution

1. We find own numbers of a matrix A.

We write the characteristic equation for a system as follows:

$$\det(A - \lambda I) = 0.$$

We obtain own numbers of the matrix A:

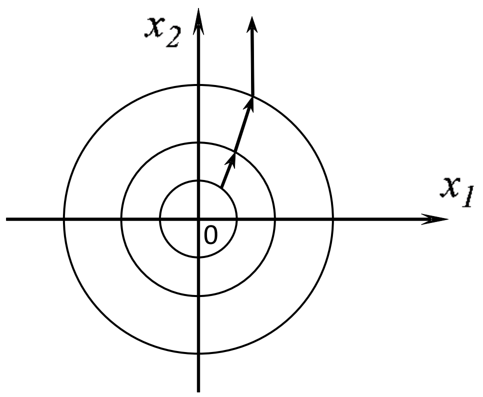
$$\det\begin{vmatrix} (-4-\lambda) & 5 \\ 1 & (-\lambda) \end{vmatrix} = 0;$$

$$(4+\lambda)\lambda - 5 = \lambda^2 + 4\lambda - 5 = 0$$

$$\lambda_1 = -5; \quad \lambda_2 = 1.$$

Hence the movement of the researched dynamic system is unstable across Lyapunov as a real part of the second root is positive, i.e. $\lambda_2 > 0$.

Geometrical interpretation:



I. Check on observability by R. Kallman's criterion

Algorithm and solution

1. The dynamic system is described in state-space by the system of the equations (*), where the matrixes:

$$A = \begin{vmatrix} -4 & 5 \\ 1 & 0 \end{vmatrix}, \quad B = \begin{vmatrix} -5 \\ 1 \end{vmatrix}.$$

2. The block matrix of observability will write down for this system as follows

$$K_{ob}^T = \left(C^T, A^T C^T\right).$$

3. We define a rank of a block matrix of observability:

$$A^T C^T = \begin{vmatrix} -4 & 1 \\ 5 & 0 \end{vmatrix} \begin{vmatrix} 1 \\ -1 \end{vmatrix} = \begin{vmatrix} -5 \\ 5 \end{vmatrix}; \; K_{ob}^T = \begin{vmatrix} 1 & -1 \\ -5 & 5 \end{vmatrix}; \; \Delta_1 = 1 \neq 0; \; \Delta_2 = 0;$$

$rank\, K_{ob}^T = 1 \neq n$.

Hence, the researched system is not observable by R. Kallman's criterion because the rank of the block matrix of observability is not equal to order of system.

II. Check on controllability by E. Gilbert's criterion

Algorithm and solution

1. The dynamic system is described in state-space by the system of the equations (*). We write down the description of a system in canonical (or diagonal) a form:

$$\begin{cases} \dot{X}^* = \Lambda X^* + B^* U \\ Y^* = C^* X^* \end{cases}. \qquad (**)$$

There are $\Lambda = V^{-1} A V$, $B^* = V^{-1} B$, $C^* = CV$.

The matrix Λ is scalar matrix which have own numbers of a matrix of A on diagonal. Hence, a scalar matrix Λ is equal:

$$\Lambda = \begin{vmatrix} -5 & 0 \\ 0 & 1 \end{vmatrix}.$$

2. We define own matrixes of a vector of V_i $\forall$ $i=1,n$ from the following identity:

$$\lambda_i V_i = A V_i,,$$

$$V = |V_1 V_2| = \begin{vmatrix} v_{11} & v_{21} \\ v_{12} & v_{22} \end{vmatrix}.$$

for i=1:

$$-5 \begin{vmatrix} v_{11} \\ v_{12} \end{vmatrix} = \begin{vmatrix} -4 & 5 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} v_{11} \\ v_{12} \end{vmatrix}.$$

We will write in a scalar form:

$$\begin{cases} -5v_{11} = -4v_{11} + 5v_{12} \\ -5v_{12} = v_{11} \end{cases}; \text{ let } v_{12}=1, \text{ then } v_{11}= -5.$$

Hence,

$$V_1 = \begin{vmatrix} -5 \\ 1 \end{vmatrix};$$

for i=2 write down: $\begin{vmatrix} v_{21} \\ v_{22} \end{vmatrix} = \begin{vmatrix} -4 & 5 \\ 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} v_{21} \\ v_{22} \end{vmatrix}$; $\begin{cases} v_{21} = -4v_{21} + 5v_{22} \\ v_{22} = v_{21} \end{cases}$;

let $v_{21}=1$, then $v_{22}= 1$.

hence,

$$V_2 = \begin{vmatrix} 1 \\ 1 \end{vmatrix}.$$

We write down of the matrix of own vectors:

$V = \begin{vmatrix} -5 & 1 \\ 1 & 1 \end{vmatrix}$; determinant *det V= -6 ≠0 is not equal to zero, therefore, there is an inverse matrix* V^{-1}.

3. We carry out check of observability by E. Gilbert's criterion:

$$C^* = CV = \begin{vmatrix} 1 & -1 \end{vmatrix} \begin{vmatrix} -5 & 1 \\ 1 & 1 \end{vmatrix} = \begin{vmatrix} -6 & 0 \end{vmatrix};$$

Hence, $y^* = -6x_1^*$.

Hence, the researched system unobservable by E. Gilbert's criterion as the matrix of C^* contains a zero column; also output coordinate of y^* does not contain full state-vector x^* (is absent x_2^*).

General conclusion: The moving of the researched system is unstable across Lyapunov and this system is unobservable by R. Kallman and E. Gilbert's criteria.

Task Investigate a dynamic system on observability by R. Kallman and E. Gilbert's criteria if the mathematical description of a system is given in the state-space in the following look:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases},$$

where matrix *A*, *B*, *C* are matrixes with constant coefficients (on variants).

Variants:

1)

$$A=\begin{vmatrix}1 & -1\\ 7 & 9\end{vmatrix},\ B=\begin{vmatrix}-3\\ 0\end{vmatrix},\ C=\begin{vmatrix}1\\ -3\end{vmatrix}.$$

2)

$$A=\begin{vmatrix}2 & 6\\ 8 & 4\end{vmatrix},\ B=\begin{vmatrix}5\\ -2\end{vmatrix},\ C=\begin{vmatrix}-1\\ 2\end{vmatrix}.$$

3)

$$A=\begin{vmatrix}-5 & 4\\ -2 & -2\end{vmatrix},\ B=\begin{vmatrix}7\\ 1\end{vmatrix},\ C=\begin{vmatrix}3\\ -2\end{vmatrix}.$$

4)

$$A=\begin{vmatrix}-8 & -4\\ -2 & -6\end{vmatrix},\ B=\begin{vmatrix}-2\\ 9\end{vmatrix},\ C=\begin{vmatrix}0\\ -2\end{vmatrix}.$$

5)

$$A=\begin{vmatrix}7 & 2\\ 4 & 5\end{vmatrix},\ B=\begin{vmatrix}2\\ 1\end{vmatrix},\ C=\begin{vmatrix}0\\ 1\end{vmatrix}.$$

6)

$$A=\begin{vmatrix}7 & 9\\ 6 & 4\end{vmatrix},\ B=\begin{vmatrix}-1\\ 0\end{vmatrix},\ C=\begin{vmatrix}2\\ 0\end{vmatrix}.$$

7)

$$A=\begin{vmatrix}5 & 6\\ 8 & 7\end{vmatrix},\ B=\begin{vmatrix}1\\ 3\end{vmatrix},\ C=\begin{vmatrix}2\\ -1\end{vmatrix}.$$

8)

$$A=\begin{vmatrix}9 & 9\\ 2 & 6\end{vmatrix},\ B=\begin{vmatrix}-3\\ 0\end{vmatrix},\ C=\begin{vmatrix}1\\ -3\end{vmatrix}.$$

9)

$$A=\begin{vmatrix}2 & -1\\ -1 & 2\end{vmatrix},\ B=\begin{vmatrix}4\\ -3\end{vmatrix},\ C=\begin{vmatrix}-1\\ 3\end{vmatrix}.$$

10)

$$A=\begin{vmatrix}10 & 11\\ 14 & 13\end{vmatrix},\ B=\begin{vmatrix}-1\\ 1\end{vmatrix},\ C=\begin{vmatrix}1\\ 2\end{vmatrix}.$$

11)

$$A=\begin{vmatrix}3 & 4\\ 6 & 5\end{vmatrix},\ B=\begin{vmatrix}1\\ -1\end{vmatrix},\ C=\begin{vmatrix}0\\ 1\end{vmatrix}.$$

12)

$$A=\begin{vmatrix}-5 & 2\\ 4 & -7\end{vmatrix},\ B=\begin{vmatrix}-1\\ 2\end{vmatrix},\ C\ =\begin{vmatrix}-1\\ 2\end{vmatrix}.$$